Analyzing the Frequency Response Function (FRF) of Motion Control Systems

Boaz Kramer

ACS-Tech80

Well behaved motion control systems are characterized by crisp movement, low tracking error and short settling times. In order to achieve these goals, relatively high bandwidths with sufficient stability margins are required.

These motion control system dynamics can be characterized by the Frequency Response Function, or FRF. Using the FRF to monitor and analyze the effects of mechanical resonances is an important step in the enhancement of system bandwidth while avoiding system stability problems.

Today's advanced motion controllers include internal Digital Signal Analyzers (DSA) that make frequency response measurements possible.

The DSA together with an FRF analyzer take system tuning and optimization a great leap forward.

The **SPiiPlus FRF Analyzer** from **ACS-Tech80** provides a good example of the implementation of this system tuning and optimization technique.

With this tool it is possible to:

- Measure the frequency response of a mechanical system
- Identify and compensate for mechanical resonances
- Increase machine bandwidth, thus improving performance
- Improve machine stability
- Compare between different machines

The remainder of this paper discusses FRF measurement, analysis, and control algorithm design.

Frequency Response Function

FRF represents the steady state transfer function of a dynamic system and describes the relation between an input and an output as a function of frequency in terms of gain and phase.

Figure 1 illustrates a general motion control system, composed of a plant and a controller.

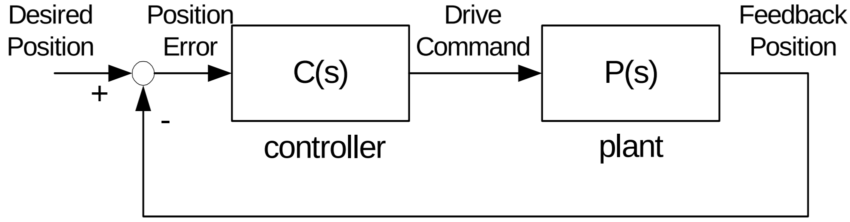

Figure 1: General Motion Control Scheme

In this article we refer to four types of FRF:

- **Plant FRF**- Plant FRF describes the relation between the drive command and the feedback position, illustrated by P(s) in Figure 1. The plant is external to the motion controller and constitutes of the mechanical stage, the motor, the feedback device and the drive.
- **Controller FRF** – Controller FRF describes the relation between the position error and the drive command, illustrated by C(s) in Figure 1. This is the control algorithm that is implemented in the motion controller.
- **Open loop FRF** – Open Loop FRF is the loop transfer function described by the function OL(s) = C(s)*P(s) as illustrated in Figure 1. This function plays a major role in a system stability analysis.
- **Closed loop FRF** - Closed loop FRF describes the relation between the desired position and the feedback position in a closed loop system and is described by the function CL(s) = C(s)*P(s)/(1+C(s)*P(s)).

Although there a number of formats in which FRF information can be presented, this paper relates to the most accepted method – the Bode diagram. The SPiiPlus FRF Analyzer offers other forms of FRF display, like the Nyquist diagram.

A Bode diagram constitutes of two parts:

- One shows the gain on a logarithmic scale or linear dB scale as a function of frequency, which is also on a logarithmic scale
- The second shows the phase on a linear scale as a function of frequency which is also on a logarithmic scale

FRF Measurement and Design

The most convenient way to measure the FRF is to excite the motion control system with a continuous sine signal with a varying frequency, often referred to as a continuous sine sweep. When the system includes a current drive, the excitation signal is added to the drive command, as illustrated in Figure 2. A set of measurement parameters needs to be specified, and includes the start frequency, the end frequency, the excitation level and the measurement resolution. The number of points per frequency decade determines the measurement resolution.

Various FRFs can be directly measured, depending on the input and output signals, as summarized in Table 1.

Table 1: FRF Measurement Criteria

FRF Type	Closed-loop FRF	Open loop FRF	Plant FRF	Controller FRF
Input Signal	Sine Sweep (n)	Drive Command (u)	Drive Command (u)	Position Error (x)
Output Signal	Controller Output (y)	Controller Output (y)	Position Error (x)	Controller Output (y)

The advantage of these measurement methods is that the system is under closed-loop control. When under closed-loop control the system is prevented from drifting into the hard-stops. It also allows controlled system motion during the FRF measurement.

FRF measurements should usually be made when the machine is in motion. If the machine is at rest, stiction may be the primary physical effect rather than the effect of load inertia.

Before FRF measurement can be made, the system has to be tuned. Since system characteristics are not yet determined, it is only required to have a very stable, low-gain, low-bandwidth servo.

Another advantage of the direct measurement method is that once the Controller FRF is known along with one of the other three FRFs (Closed-loop FRF, Open-loop FRF or Plant FRF), the remaining FRFs can be calculated without any further measurements.

There is a slight disadvantage with this method because Closed-loop FRF measurement does not take into account feed forward compensations that are directly added to the drive command. Acceleration feed forward is one example. However, these compensations have no effect on the stability of the system.

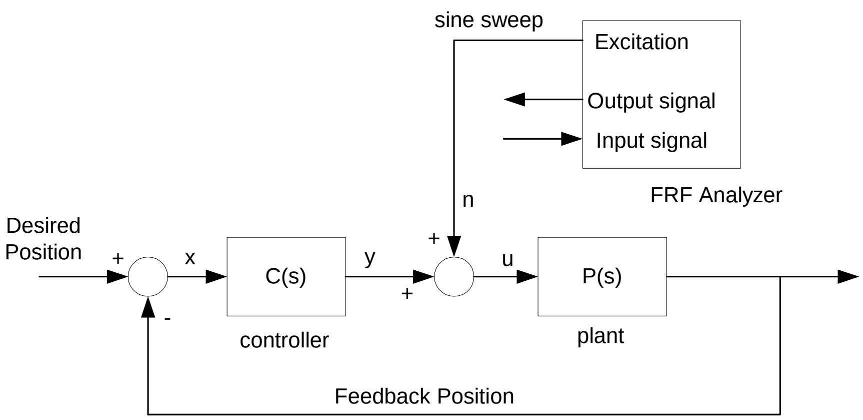

Figure 2: FRF Measurement in a Closed-Loop

The following sections describe the Plant FRF, Controller FRF and Open-loop FRF.

Plant FRF

Figure 3 illustrates a typical Plant FRF.

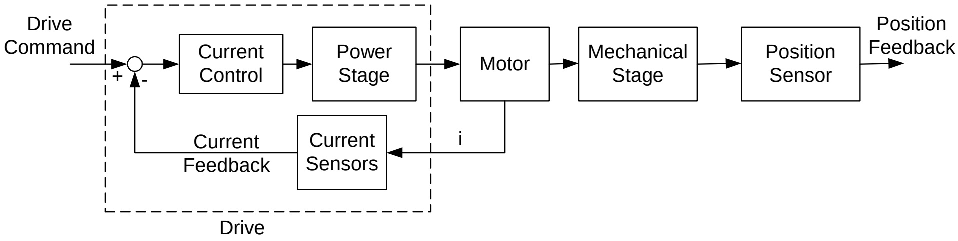

Figure 3: Typical Plant

A plant is typically composed of the following components:

- **Drive** – The drive includes a power stage and, typically, a closed current loop. A low-pass filter is usually used to filter the current feedback. The entire transfer function of the current controlled drive can be considered as a low-pass filter.
- **Motor** –The motor produces torque or force in case of a linear motor.
- **Mechanical stage or load**
- **Position Sensor** – the position sensor is typically equipped with a low-pass filter.

The Plant FRF describes the relation between the drive command and the position feedback.

Figure 4 illustrates the FRF measurement of a simple rotary servo motor displayed in a Bode diagram format. This measurement was made with ACS-Tech80's SPiiPlus FRF Analyzer.

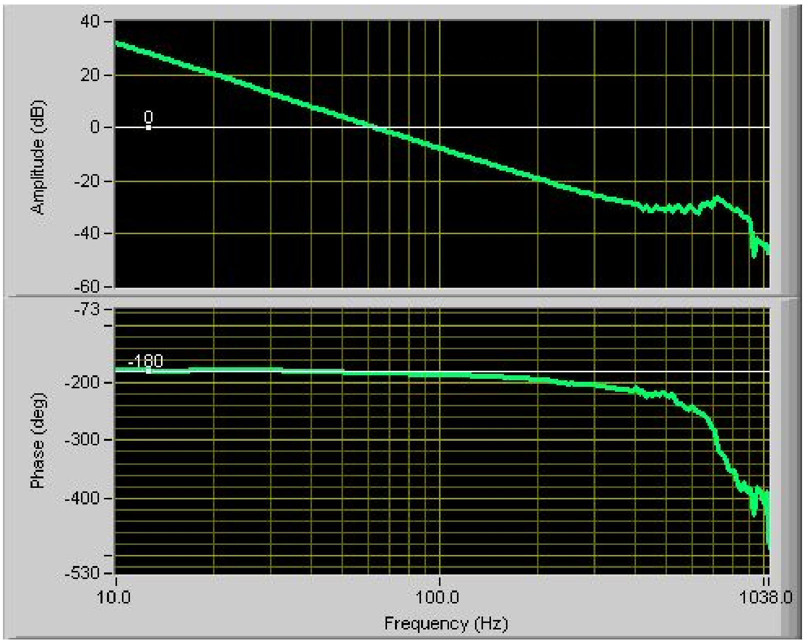

Figure 4: Simple Motor FRF

At low frequencies, the typical behavior of a double integrator is observed: the gain has –40 dB per decade slope, while the phase is -180^0. As frequency increases, additional phase lag is added because of the limited bandwidths of the drive and other components in the system. A slight resonance is observed close to 700 Hz. This is due to a compliant coupling between the motor and its encoder.

Controller FRF

Figure 5 illustrates the most common motion control algorithm and includes a position loop and a velocity loop. The velocity loop includes a PI filter, an optional 2^{nd} order low-pass filter and an optional notch filter. The position loop includes a proportional gain and a velocity feed forward is added to the output of the position loop.

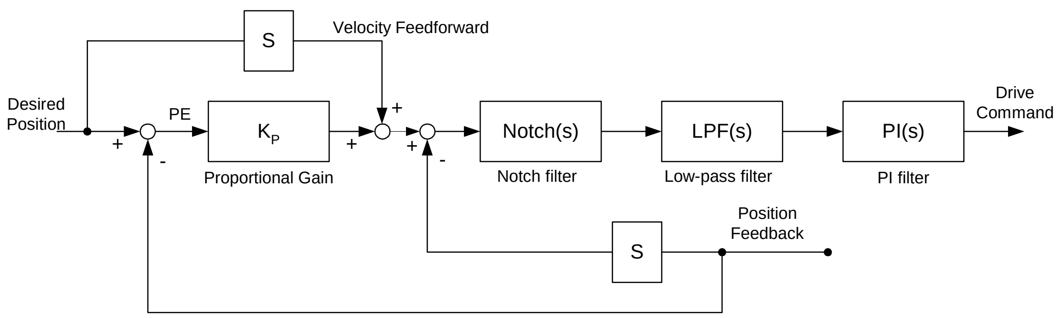

Figure 5: Basic Motion Control Scheme

For frequency response purposes, it is more convenient to look at an equivalent control scheme, as illustrated in Figure 6.

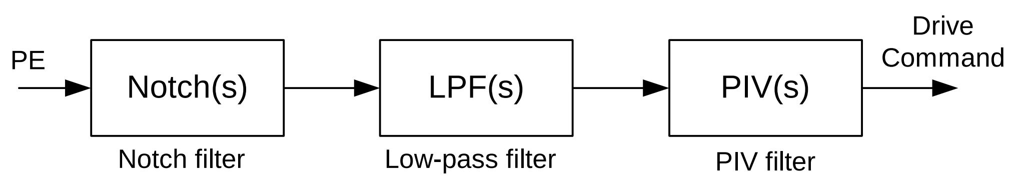

Figure 6: Equivalent Control Scheme

The so-called "PIV filter" in Figure 6 has a structure similar to a PID filter, as shown in the following formula:

$$\mathrm{PIV(s)} = K_V \times \frac{(s + K_P)(s + K_I)}{s}$$

This FRF of this filter is governed by three parameters:

1. K_V – proportional gain of the velocity loop
2. K_I - integrator gain of the velocity loop
3. K_P - proportional gain of the position loop

The filter consists of a single pole at zero frequency (an integrator), and two zeros whose locations are determined by the K_P and K_I parameters. The first zero indicates the end of the integrator action. The second zero indicates the beginning of the differential action, which is necessary to achieve a stable, non-oscillating system.

The low pass filter in series with the PIV helps to attenuate high frequency resonances.

Figure 7 illustrates a typical controller FRF. The typical behavior of the PIV filter in series with a second order law pass filter is clearly observed.

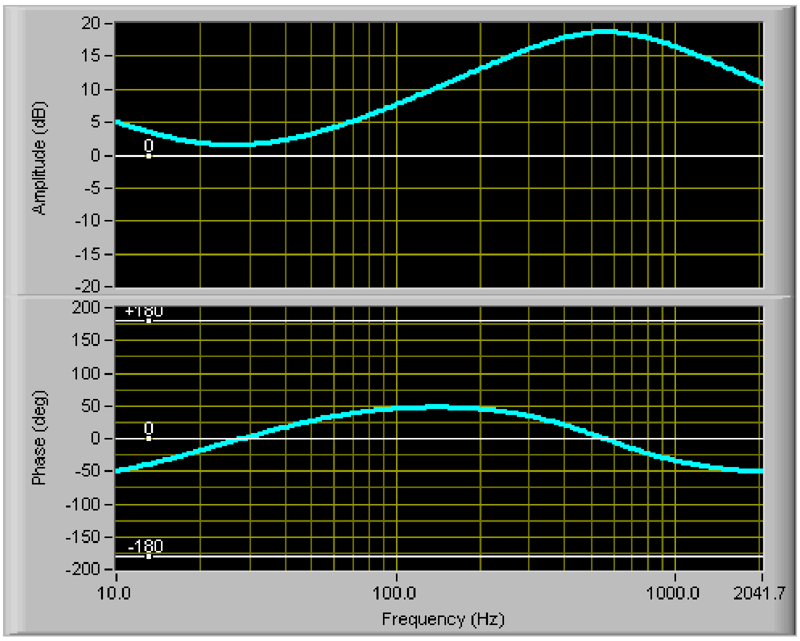

Figure 7: Controller FRF: PIV in Series with 2nd Order Low Pass Filter

The Controller FRF can also include notch filters, which are sometime used to suppress resonances. It is not guaranteed that notch filters will suppress the negative effects of resonances on stability. For example, if for several similar systems, the frequency of the resonance is not the same; a notch with fixed settings will not be suitable for all such systems.

One of the great advantages of the SPiiPlus FRF Analyzer is that it is possible to modify the controller FRF, change the servo settings, compensate resonances and simulate the effect on various FRFs, without to re-measuring the system.

Open Loop FRF

Open loop FRFs are the product of the Plant FRF and the Controller FRF.

Figure 8 illustrates a typical Open Loop FRF for a simple servo motor example.

It is possible to distinguish three frequency regions, separated by a -180^0 crossing in the phase diagram. At low frequencies the phase is typically below -180^0 due to the integrator action.

In the medium frequency range the phase lies above -180^0 due to the differential action. At high frequencies the phase falls again below -180^0, because of the 2nd order low-pass filter and because of the limited bandwidth of other various components such as the drive.

The Open-loop FRF is extremely important for the stability analysis of the system.

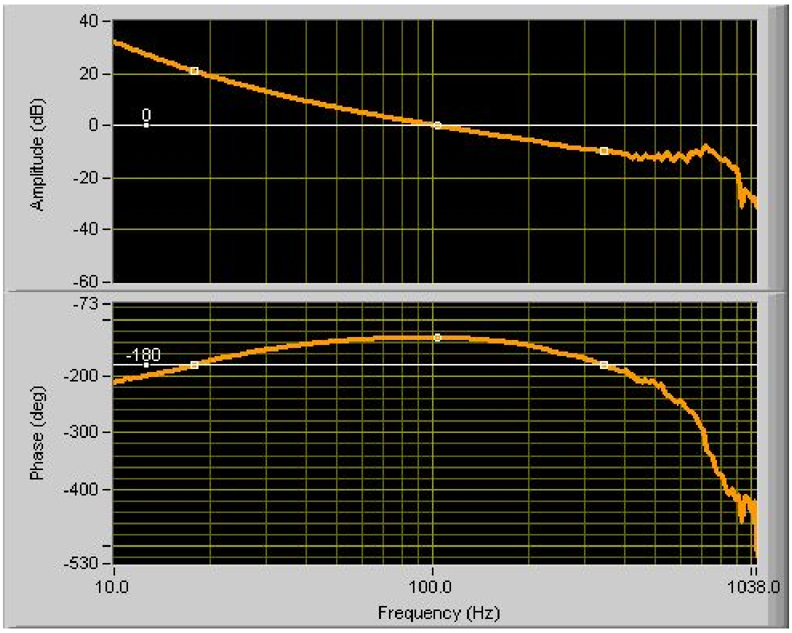

Figure 8: Open-Loop FRF

Closed-Loop FRF

A Closed-loop FRF illustrates the relation between the desired position and the position feed back. If the output follows the input command with no error, the FRF would be ‘1’, with 0 dB gain and 0^0 phase.

Figure 9 illustrates a Closed loop FRF which corresponds to this simple motor example

From the closed loop FRF it is possible to quantify the closed loop bandwidth. The traditional definition of bandwidth is the frequency at which the closed loop FRF magnitude equals –3dB, or the phase equals -90^0, whichever comes first.

However, this definition can often be over-optimistic due to high sensitivity to the presence of resonances. An alternative definition is the first gain crossover frequency of the open loop FRF. This definition provides a direct indication of the bandwidth, even in the presence of resonances.

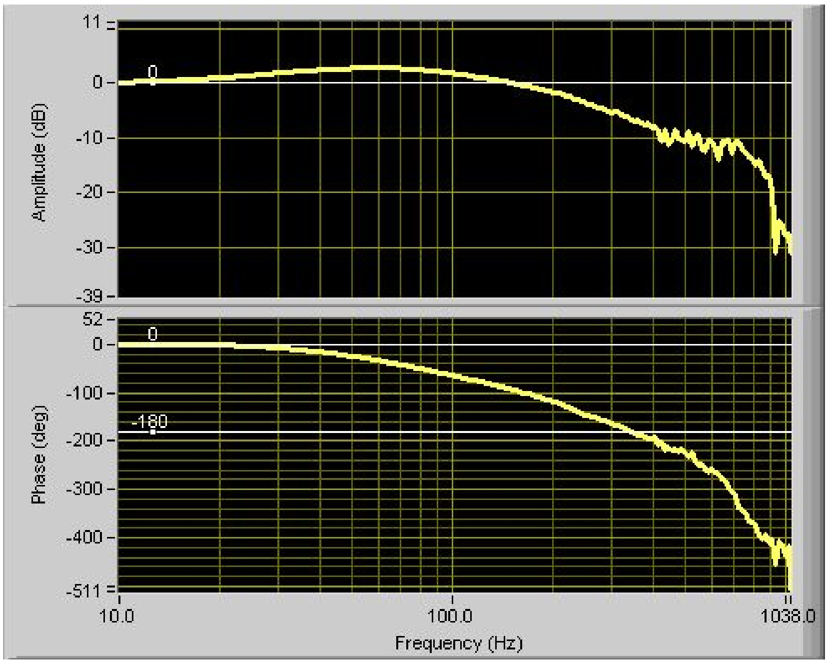

Figure 9: Closed Loop FRF

Stability Analysis

Instability occurs when the feedback becomes positive. This situation is characterized by a phase shift of -180^0 and amplitude of 1 (0 dB).

The level of stability is typically judged at certain frequencies, known as crossover frequencies, as follows:

- **Gain Crossover Frequency-** 0 dB crossing of the Open loop FRF gain.
- **Phase Crossover Frequency-** (+/-180) degrees crossing of the Open-loop FRF phase.

Two criteria quantify stability level:

- **Gain Margin (GM)** – GM measures how much the gain can be increased before instability results. It is equal to the amount by which the open loop gain is less than 0dB at the phase cross over frequency.
- **Phase Margin (PM)** – PM measures how much delay can be added to the loop before instability results. It is equal to the amount by which the open loop phase exceeds -180^0 at the gain cross over frequencies.

Stability Margin definitions are illustrated graphically in Figure 10.

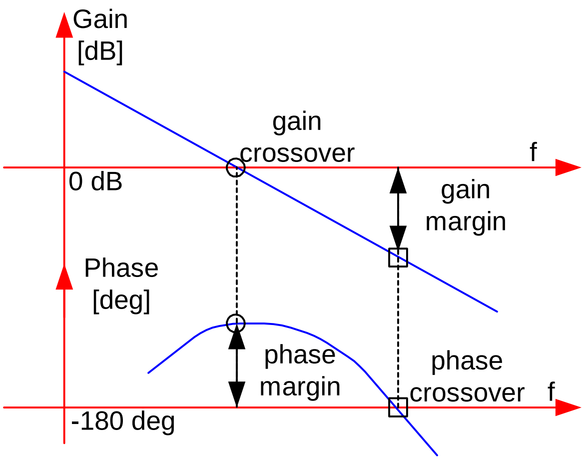

Figure 10: Stability Margin Definitions

The **SPiiPlus FRF Analyze**r displays the various crossover points. Simply click one of these intersections to show frequency, gain, phase and stability margin details.

The level of stability is also depicted in the Closed-loop FRF.

This is the Resonant Peak illustrated in Figure 11. The Resonant Peak is not caused by mechanical resonance, but rather by damping of the closed loop system and its level of stability. The higher the peak, the less stable the system. As a general rule of thumb, the resonant peak should be lower than 6dB. In Figure 9 the Resonant Peak height is about 4.5dB, indicating a good level of stability.

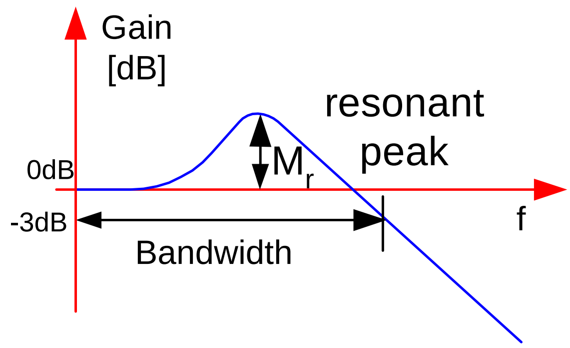

Figure 11: Resonant Peak Definition

Non-linear Effects

There are some limitations in using FRF analyses. FRF analyses relate to linear systems which are usually not the case. In fact, the FRF measurement is a linearization process. Due to non-linear effects, the coherence of an FRF measurement may be affected. For example:

- Different measurement results when the axis is idle and when the axis is moving. This might be due the non-linear effect of stiction.
- Different results for different excitation levels.
- Non-linearities due to saturations in the system, saturation of the drive or saturation of the control algorithm including integrator windup, current limit, and so on.
- Non-linear effects in the system such as backlash.
- Cross coupling between axes can also affect FRF measurement results as in the case where one axis location changes the center of gravity of a second axis.
- Noise may affect the measurement results, especially at high frequencies when the signal to noise ratio is low.

Analysis Example

Figure 12 illustrates a compliantly coupled motor and load, which is a well-known lumped-parameter model with the following parameters:

- The motor inertia is J_M.
- The equivalent load inertia is J_L.
- The spring constant of the transmission is K and the viscous damping coefficient is d.
- The motor torque is T_M, which in most cases can be considered proportional to motor current.

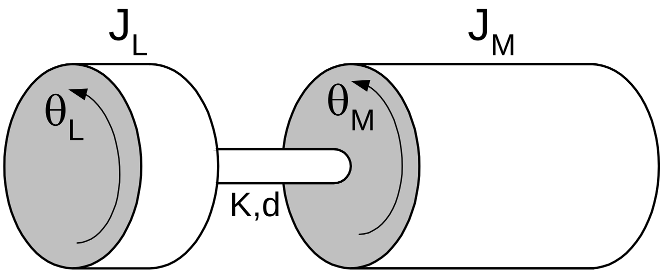

Figure 12: Compliantly Coupled Motor and Load

The dynamics of this model are straightforward and are described by the block diagram in Figure 13, where S = the Laplace operator.

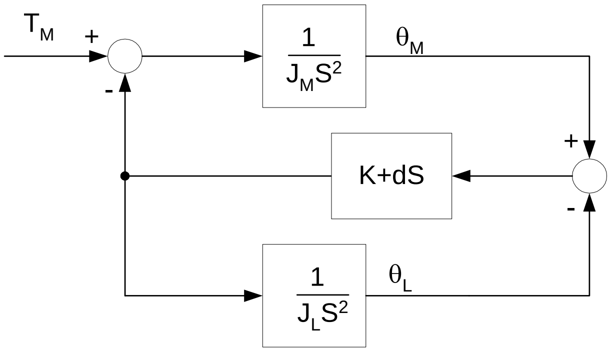

Figure 13: Compliant Coupled Motor and Load

It is interesting to observe the FRF of this system in two cases:

1. When the position sensor is mounted on the motor the FRF is the transfer function between the motor torque (T_M) and the motor position (θ_M).

2. When the position sensor is mounted on the load the FRF is the transfer function between the motor torque (T_M) and the load position (θ_L).

Each case yields different FRFs.

In the first case the plant FRF can be described as:

$$\frac{\theta_M}{T_M} = \left(\frac{1}{J_M + J_L} \frac{1}{s^2} \right) \left(\frac{J_L s^2 + ds + K}{J_L J_M / (J_L + J_M) s^2 + ds + K} \right)$$

Figure 14 shows an actual FRF measurement of this arrangement.

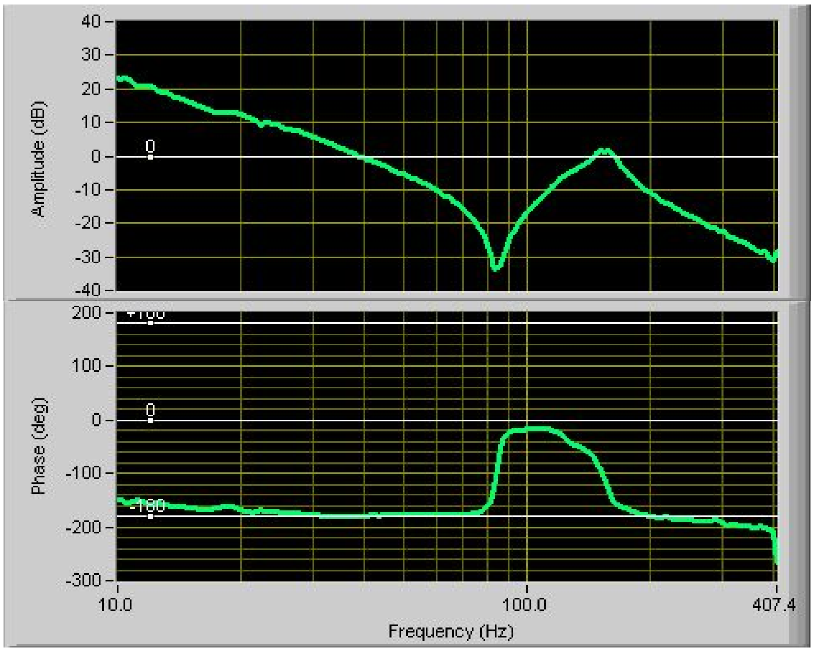

Figure 14: Plant FRF when Sensor is on the Motor

At low frequencies ($s \to 0$) the system can be considered as a rigidly coupled motor and load:

$$\frac{\theta_M}{T_M} \approx \left(\frac{1}{J_M + J_L} \frac{1}{s^2} \right)$$

This is illustrates the behavior characteristic of a double integrator.

A second-order zero (so-called "anti-resonance") is observed at the following frequency:

$$f_A = \frac{1}{2\pi} \sqrt{K / J_L}$$

This is the natural frequency of the spring and the load. At this frequency the numerator is minimized and all the energy is fed directly to the load.

A second-order pole (resonance) is observed at the following frequency:

$$f_R = \frac{1}{2\pi} \sqrt{\frac{K}{J_L J_M / (J_L + J_M)}}$$

This is the natural frequency of the spring and the motor-load combination.

At this frequency the denominator is minimized and the system gain is very high.

Note that the anti-resonant frequency is smaller than the resonant-frequency, causing a phase lead as can be observed in the phase section of the Bode diagram of figure 14.

At high frequencies ($s \to \infty$) the load is effectively decoupled from the motor and does not follow its motion -

$$\frac{\theta_M}{T_M} \approx \left(\frac{1}{J_M + J_L}\frac{1}{s^2}\right)\left(\frac{J_L s^2}{J_L J_M/(J_L + J_M)s^2}\right) = \frac{1}{J_M}\frac{1}{s^2}$$

This is again characteristic behavior of a double integrator. Here, the gain is higher as only the motor inertia is involved. In other words, the motor moves only its own inertia

The extra gain due to the decoupling of the load mass can have a negative effect on stability. This is illustrated in Figure 15 that shows an open-loop FRF of the system, with a certain set of tuning.

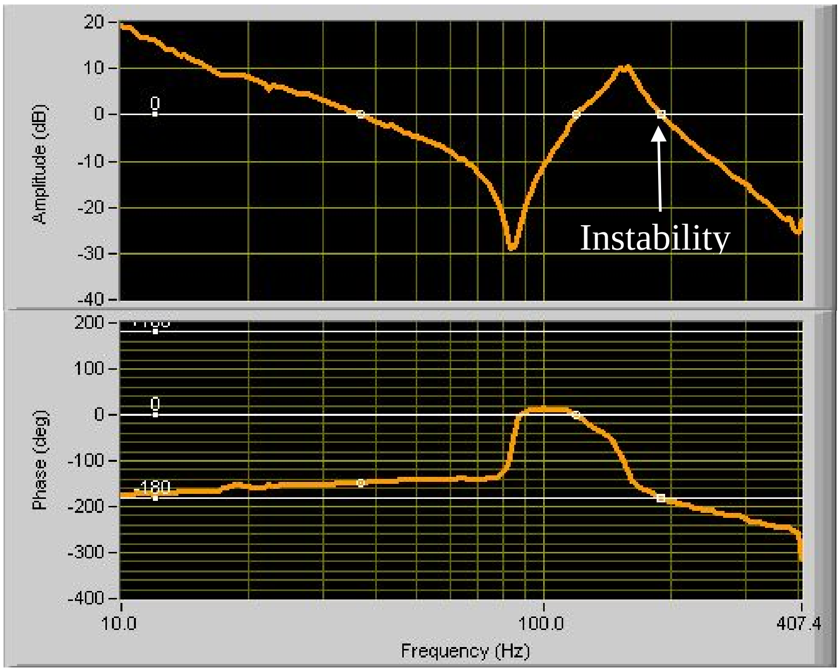

Figure 15: Unstable Open-Loop FRF

Instability occurs at the frequency where the phase is -180^0 and the gain is zero dB. In order to prevent this situation the gain must be reduced. Alternatively, a notch filter can be used to attenuate the resonance.

Note that the oscillation frequency is larger than the resonant frequency of the system. This may be quite misleading if one tries to determine the appropriate notch frequency by observing only the time-domain response.

In the second case, when the position sensor is mounted on the load, the transfer function is:

$$\frac{\theta_L}{T_M} = \left(\frac{1}{J_M + J_L}\frac{1}{s^2}\right)\left(\frac{K + ds}{J_L J_M/(J_L + J_M)s^2 + ds + K}\right)$$

Once again, at frequencies below the system resonant frequency, the system behaves practically as a rigid body. Above the resonant frequency the load is decoupled from the motor. In this example the feedback indicates the load motion. Figure 16 illustrates the Plant FRF.

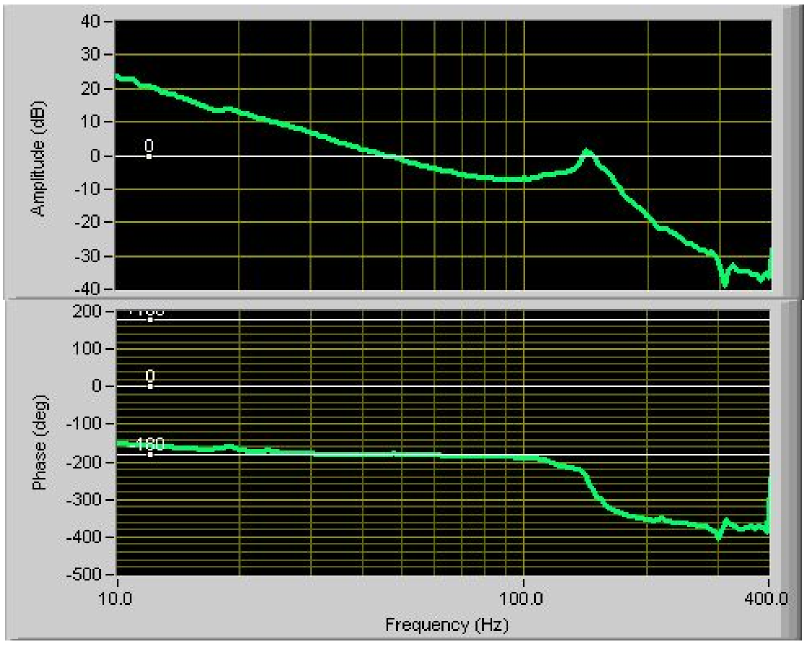

Figure 16: Plant FRF with Load Mounted Sensor

In this FRF diagram a second-order pole is evident at the resonance frequency.

The first order-zero obtained from the viscous damping of the system is usually is at high frequencies, as the damping is relatively low. As a result a significant phase lag is introduces because of the system resonance. This can have a devastating effect on stability, and the bandwidth of such system may have to be significantly reduced.

Conclusion

This discussion of FRF basics has shown that FRF measurement can be a useful tool for characterizing motion control system mechanics and resonances, and their effect on system stability.

Advanced FRF tools, such as the SPiiPlus FRF Analyzer can provide a positive contribution for control algorithm design with maximum system bandwidth while preserving adequate stability margins.